Лекция 6

Функции Габора и их применение

Dennis Gabor (1900 - 1979)

Hungarian-born electrical engineer who won the Nobel Prize for Physics in 1971 for his invention of holography, a system of lensless, three-dimensional photography that has many applications.

A research engineer for the firm of Siemens and Halske in Berlin from 1927, Gabor fled Nazi Germany in 1933 and worked with the Thomson-Houston Company in England, later becoming a British subject. In 1947 he conceived the idea of holography and, by employing conventional filtered-light sources, developed the basic technique. Because conventional light sources generally provided either too little light or light that was too diffuse, holography did not become commercially feasible until the demonstration, in 1960, of the laser, which amplifies the intensity of light waves.

In 1949 Gabor joined the faculty of the Imperial College of Science and Technology, London, where in 1958 he became professor of applied electron physics. His other work included research on high-speed oscilloscopes, communication theory, physical optics, and television.

$$(\Delta x)^2(\Delta s)^2 = \frac{\int x^2 f f^* \, dx \int s^2 F F^* \, ds}{\int f f^* \, dx \int F F^* \, ds}$$

$$= \frac{\int xf.xf^* \, dx \int f' f'^* \, dx}{4\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$\geq \frac{\left| \int (xf^*.f' + xf.f'^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{\left| \int x \frac{d}{dx} (f f^*) \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{\left| \int f f^* \, dx \right|^2}{16\pi^2 \left(\int f f^* \, dx \right)^2}$$

$$= \frac{1}{16\pi^2}.$$

Дисперсия.

$$\sigma^2 = \langle (x - \langle x\rangle)^2 \rangle = \frac{\int\limits_{-\infty}^{+\infty} (x - \langle x\rangle)^2 f(x)\mathrm{d}x}{\int\limits_{-\infty}^{+\infty} f(x)\mathrm{d}x} = -\frac{F''(0)}{4\pi^2 F(0)} + \frac{(F'(0)^2)}{4\pi^2 (F(0))^2}.$$

$$\sigma_{f*g}^2 = \sigma_f^2 + \sigma_g^2.$$

Ширина локализации.

$$W_f = \frac{\int\limits_{-\infty}^{+\infty} f(x)\mathrm{d}x}{f(0)} = \frac{F(0)}{\int\limits_{-\infty}^{+\infty} F(s)\mathrm{d}s} = \frac{1}{W_F}.$$

$$f'(x) = -2bxf(x) \Rightarrow f(x) = ae^{-bx^2}$$

Фильтры Габора

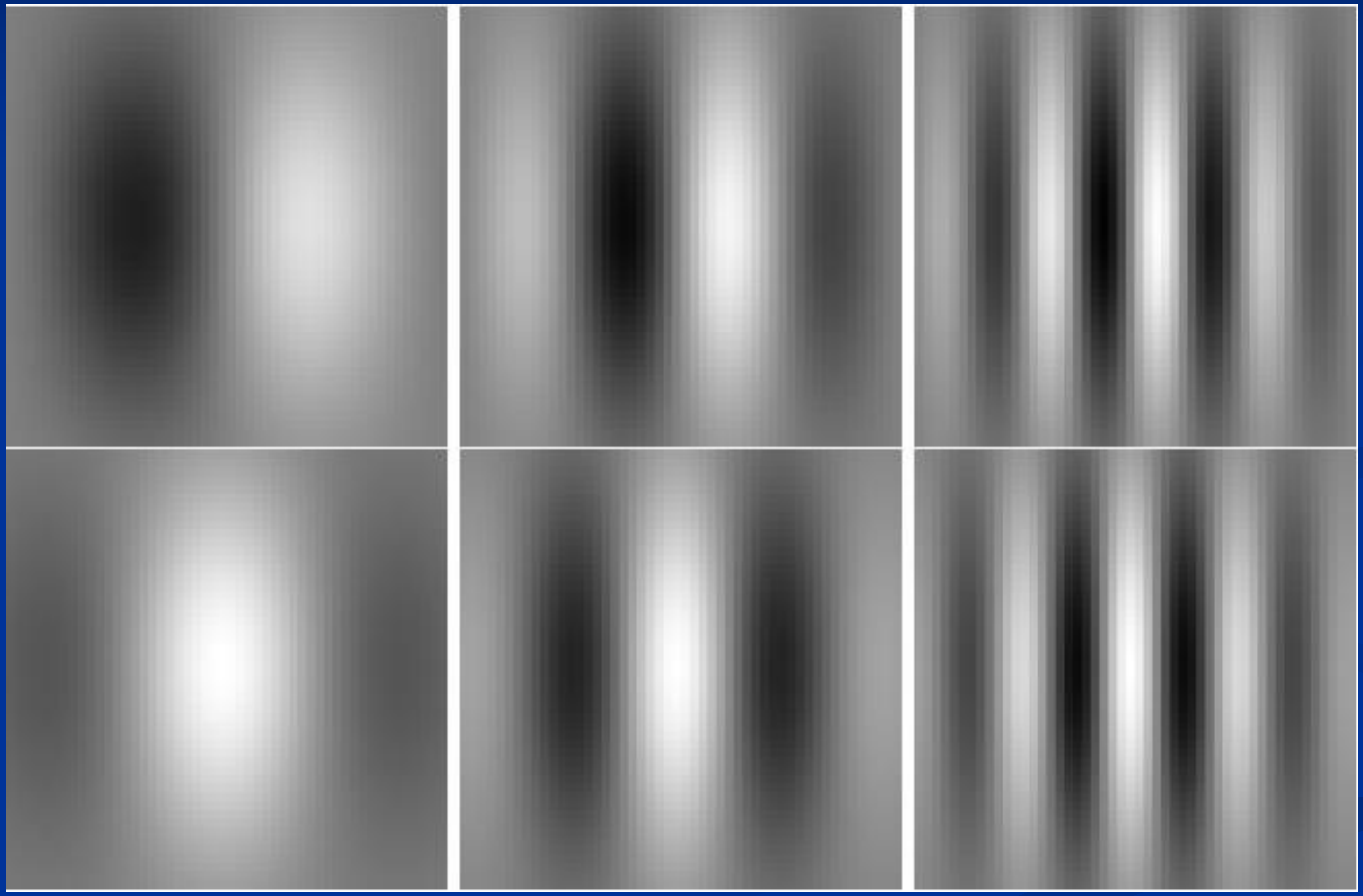
Gabor filters are formed by modulating a complex sinusoid by a Gaussian function:

$$g(x,y) = \overbrace{\frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{1}{2}\left(\frac{\tilde{x}^2}{\sigma_x^2}+\frac{\tilde{y}^2}{\sigma_y^2}\right)\right)}^{\text{gaussian envelope}} \cdot \overbrace{\exp\left(2\pi j\omega\tilde{x}\right)}^{\text{complex sinusoidal}}$$

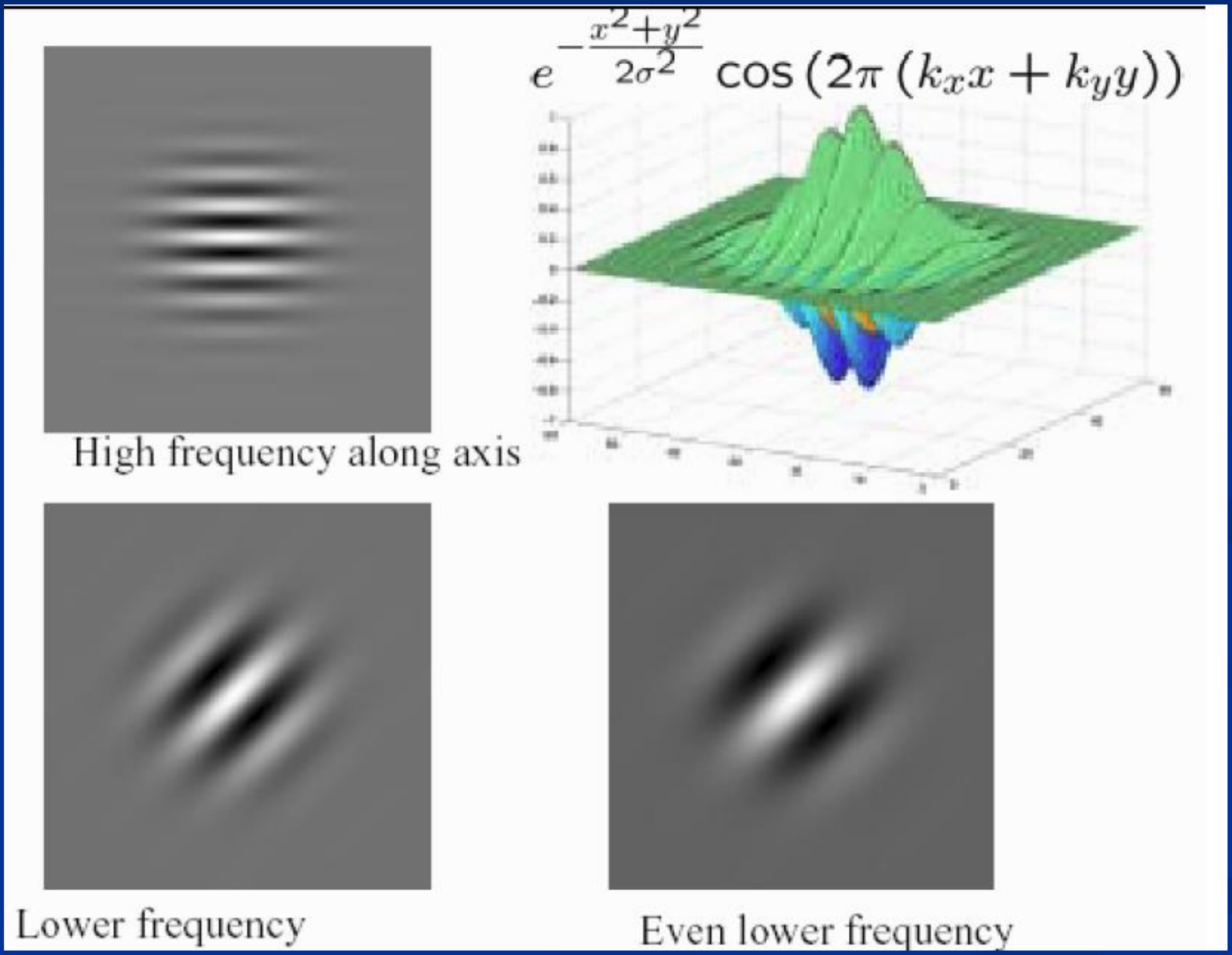
$$\text{with} \quad \begin{cases} \tilde{x} = x\cos(\theta) + y\sin(\theta) \\ \tilde{y} = -x\sin(\theta) + y\cos(\theta) \end{cases}$$

- σ_x and σ_y control spatial extent of filter
- θ is the orientation
- ω is the radial frequency of the sinusoid

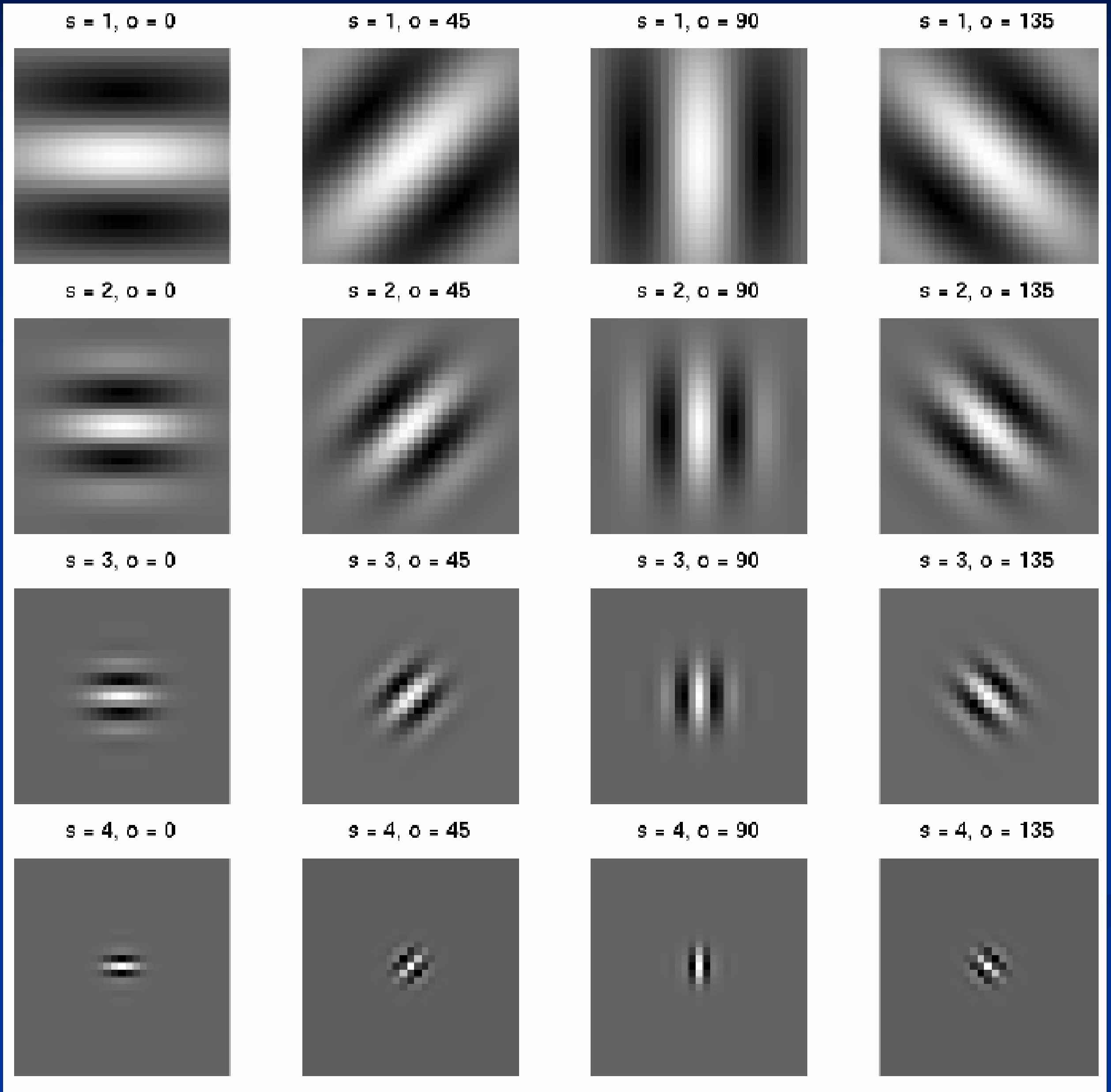
Фильтры Габора



Фильтры Габора



Фильтры Габора



Функции Габора

$$g(x, y) = \left(\frac{1}{2\pi\sigma_x\sigma_y}\right)\exp\left[-\frac{1}{2}\left(\frac{x^2}{\sigma_x^2} + \frac{y^2}{\sigma_y^2}\right) + 2\pi jWx\right],$$

$$G(u, v) = \exp\left\{-\frac{1}{2}\left[\frac{(u - W)^2}{\sigma_u^2} + \frac{v^2}{\sigma_v^2}\right]\right\},$$

where $\sigma_u = 1/2\pi\sigma_x$ and $\sigma_v = 1/2\pi\sigma_y$.

Габоровские вейвлеты

$$g_{mn}(x, y) = a^{-m} g(x', y'), \quad a > 1, \quad m, n = \text{integer}$$

$$x' = a^{-m}(x \cos\theta + y \sin\theta) \quad \text{and}$$

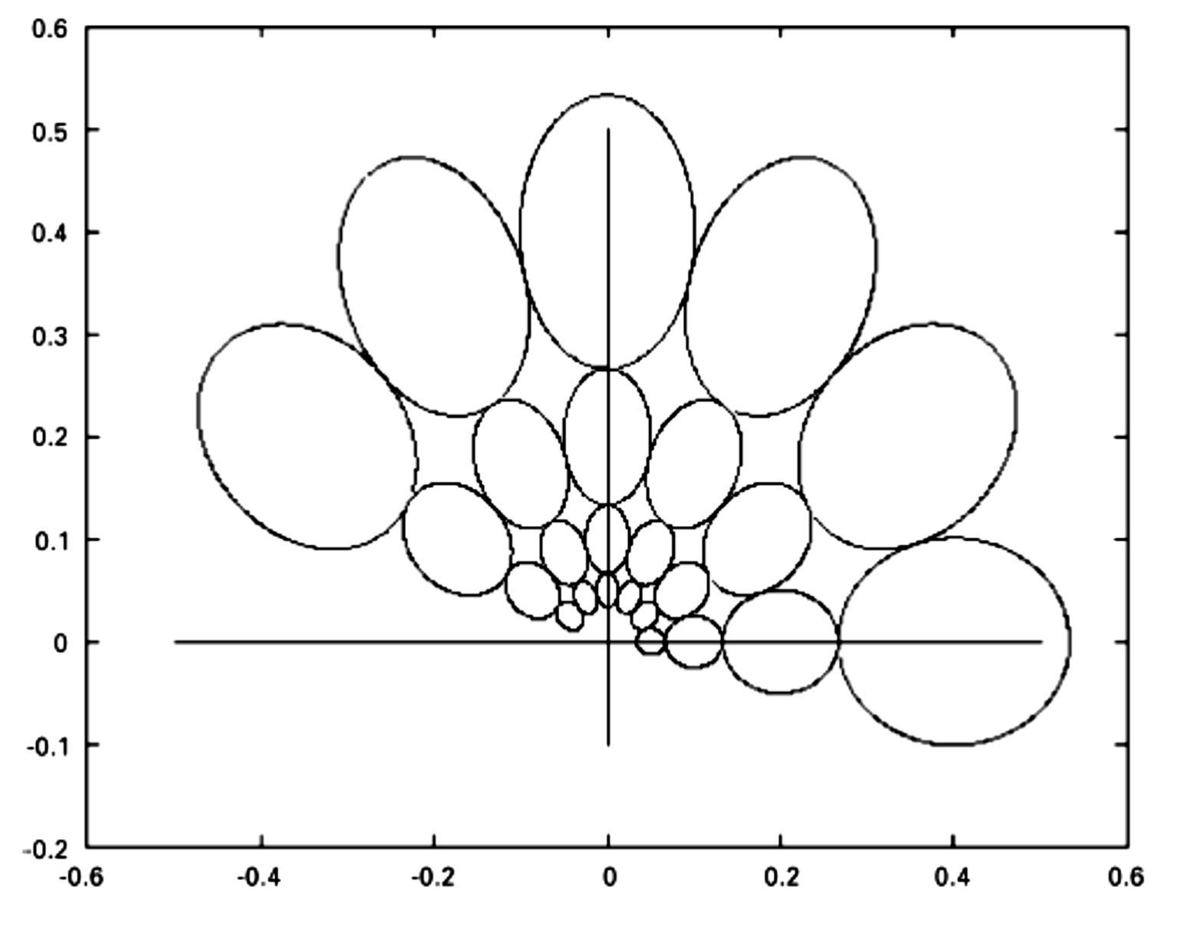
$$y' = a^{-m}(-x \sin\theta + y \cos\theta),$$

$$\theta = n\pi / K$$

K – number of orientations

Габоровские вейвлеты

$$W_{mn}(x, y) = \int I(x, y) g^*_{mn}(x - x_1, y - y_1)\, \mathrm{d}x_1\, \mathrm{d}y_1$$



$U_{\mathbf{h}} = 0.04$, $U_{\mathbf{l}} = 0.05$, $K = 6$ and $S = 4$

Габоровские вейвлеты

$$a = (U_{\mathbf{h}}/U_{\mathbf{l}})^{1/(S-1)}, \quad \sigma_u = \frac{(a-1)U_{\mathbf{h}}}{(a+1)\sqrt{2\ln 2}},$$

$$\sigma_v = \tan\left(\frac{\pi}{2k}\right)\left[U_{\mathbf{h}} - 2\ln 2\left(\frac{\sigma_u^2}{U_{\mathbf{h}}}\right)\right] \times \left[2\ln 2 - \frac{(2\ln 2)^2\sigma_u^2}{U_{\mathbf{h}}^2}\right]^{-1/2}$$

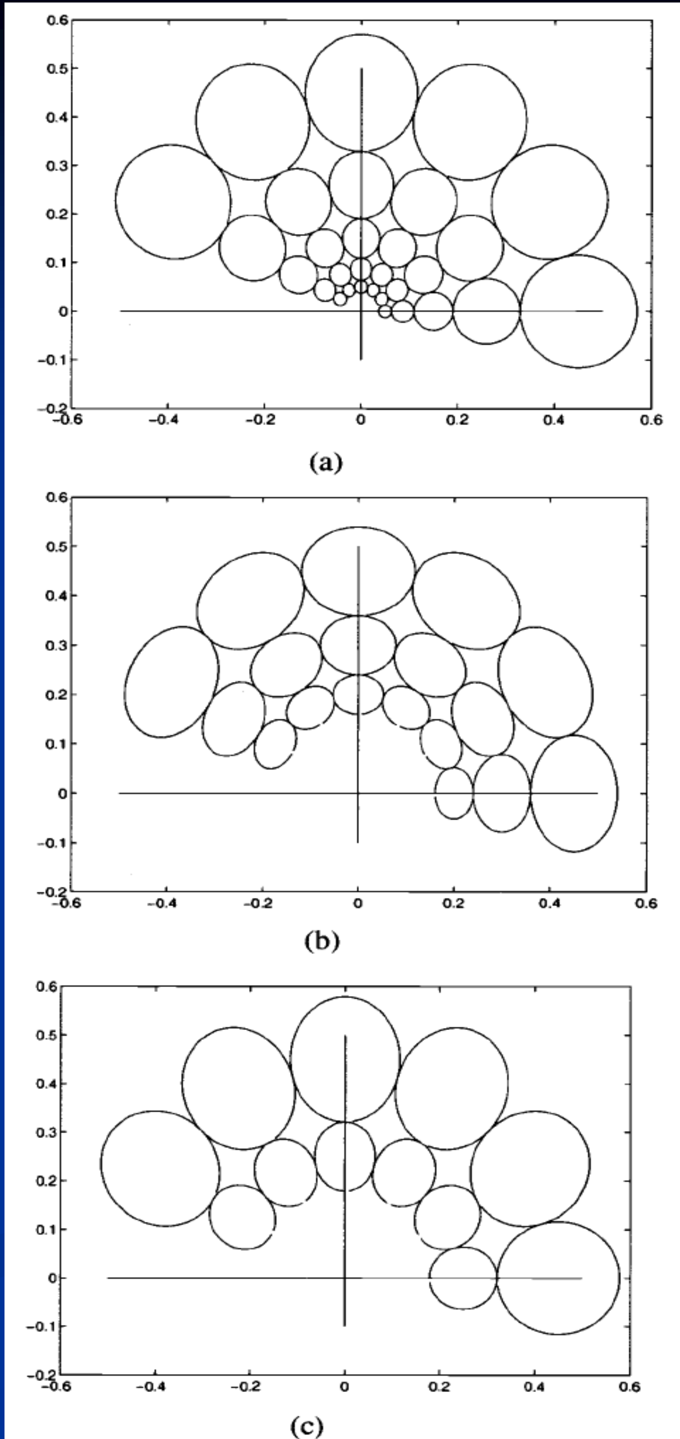
$W = U_{\mathbf{h}}$ and $m = 0, 1, \ldots, S-1$

K=6

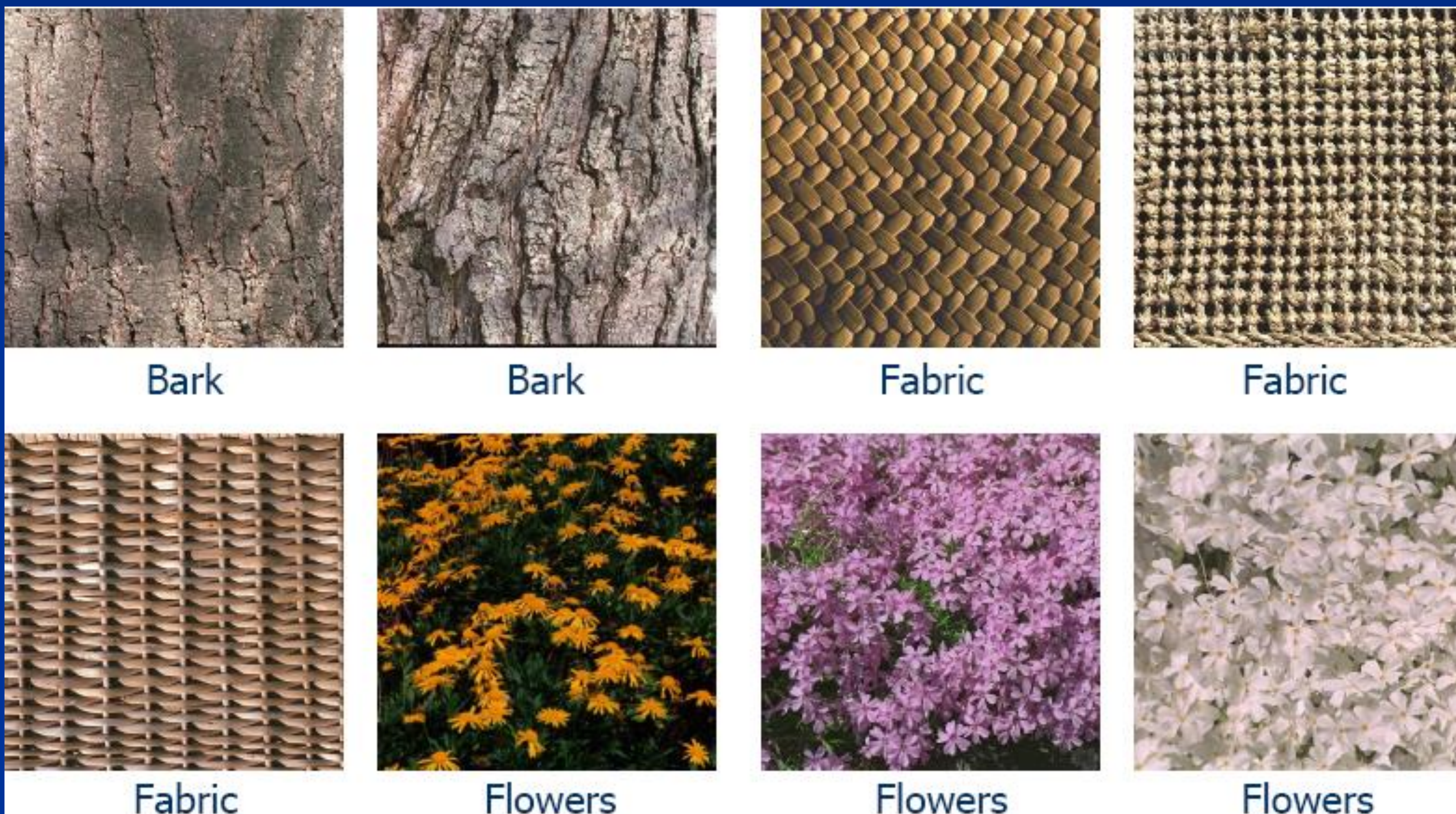
$a) \quad \sigma = 5, S = 5$

$b) \quad \sigma = 1.25, S = 3$

$c) \quad \sigma = 1, S = 2$

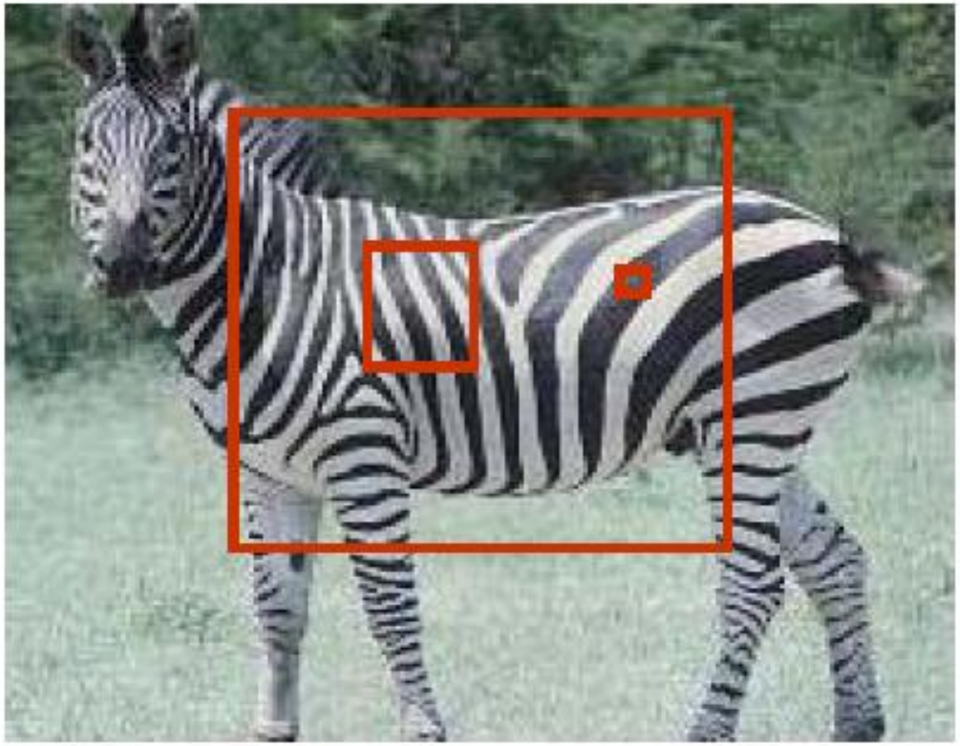


Текстуры



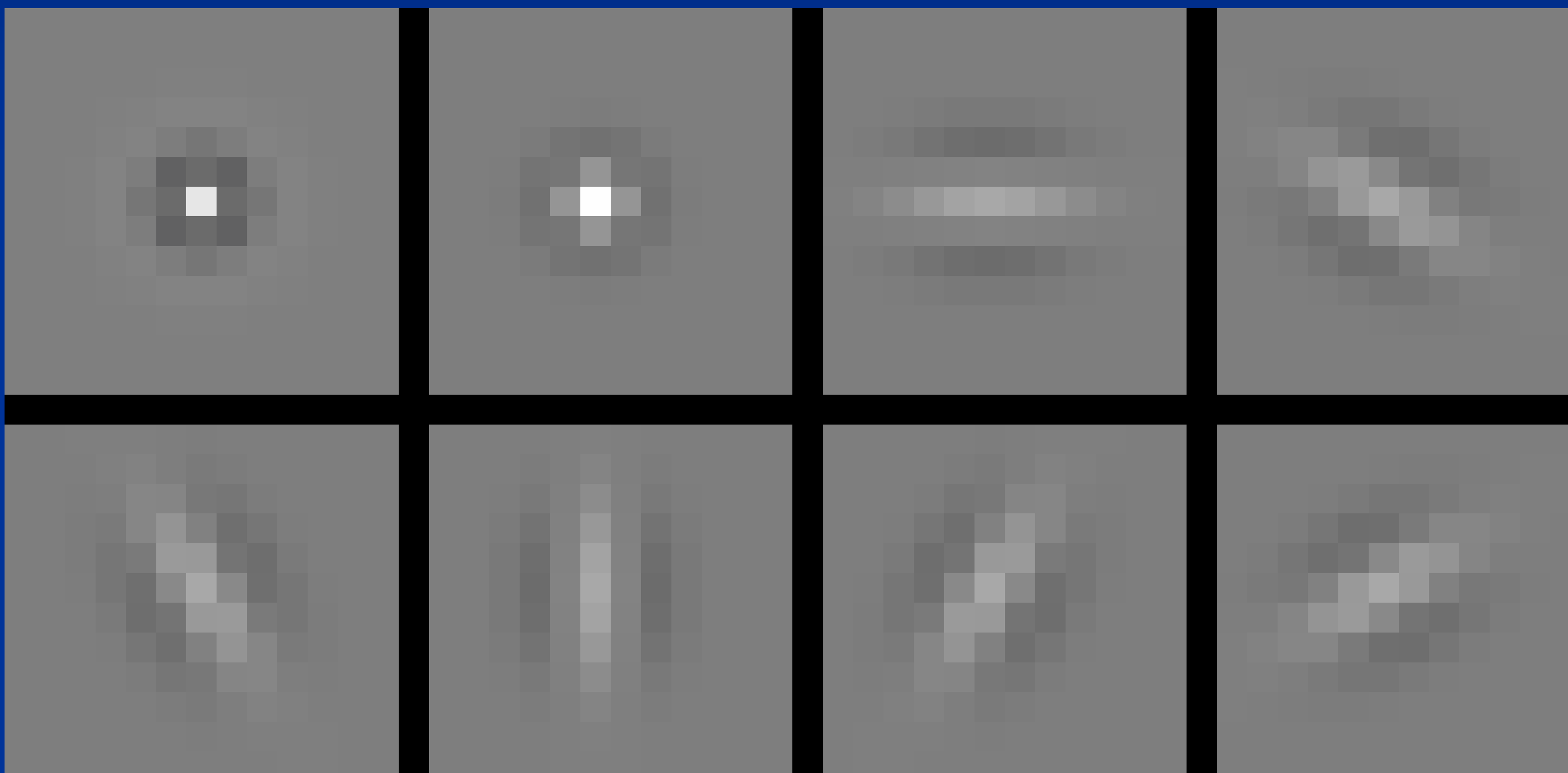
Текстуры

- Whether an effect is a texture or not depends on the scale at which it is viewed.

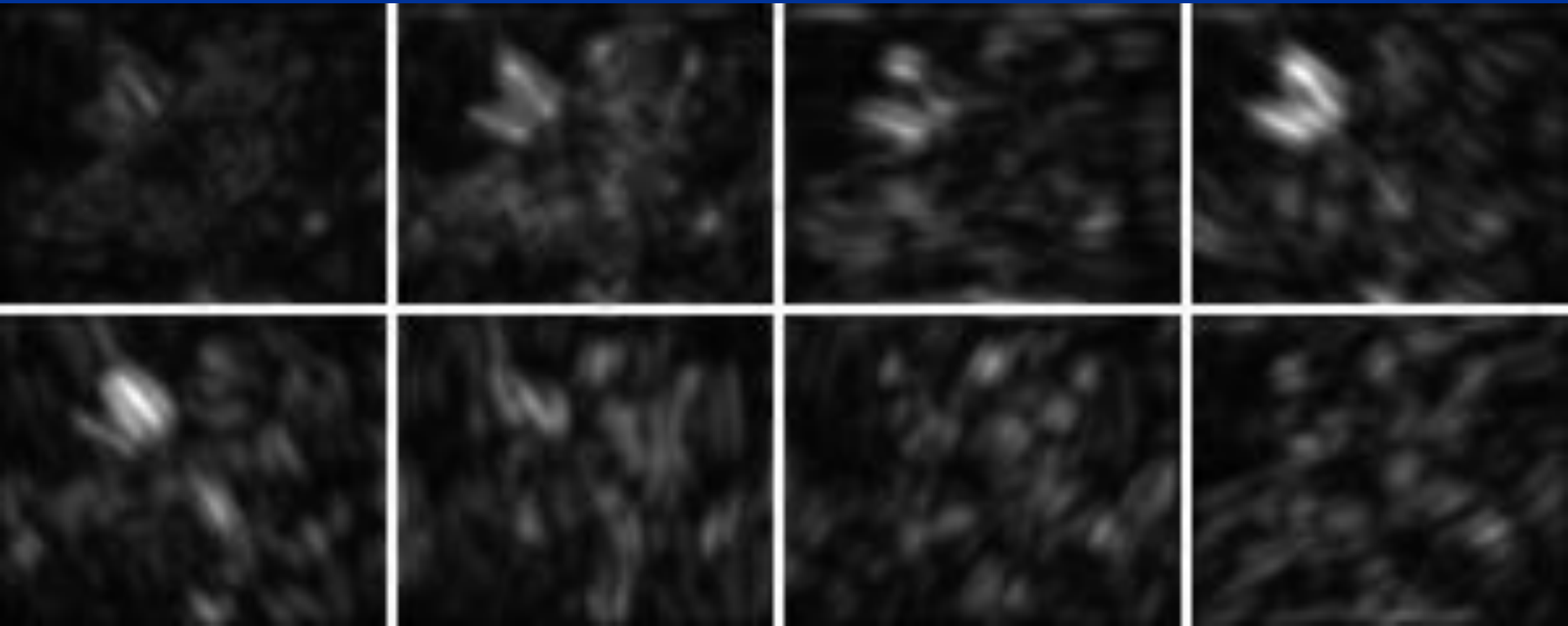


Текстуры

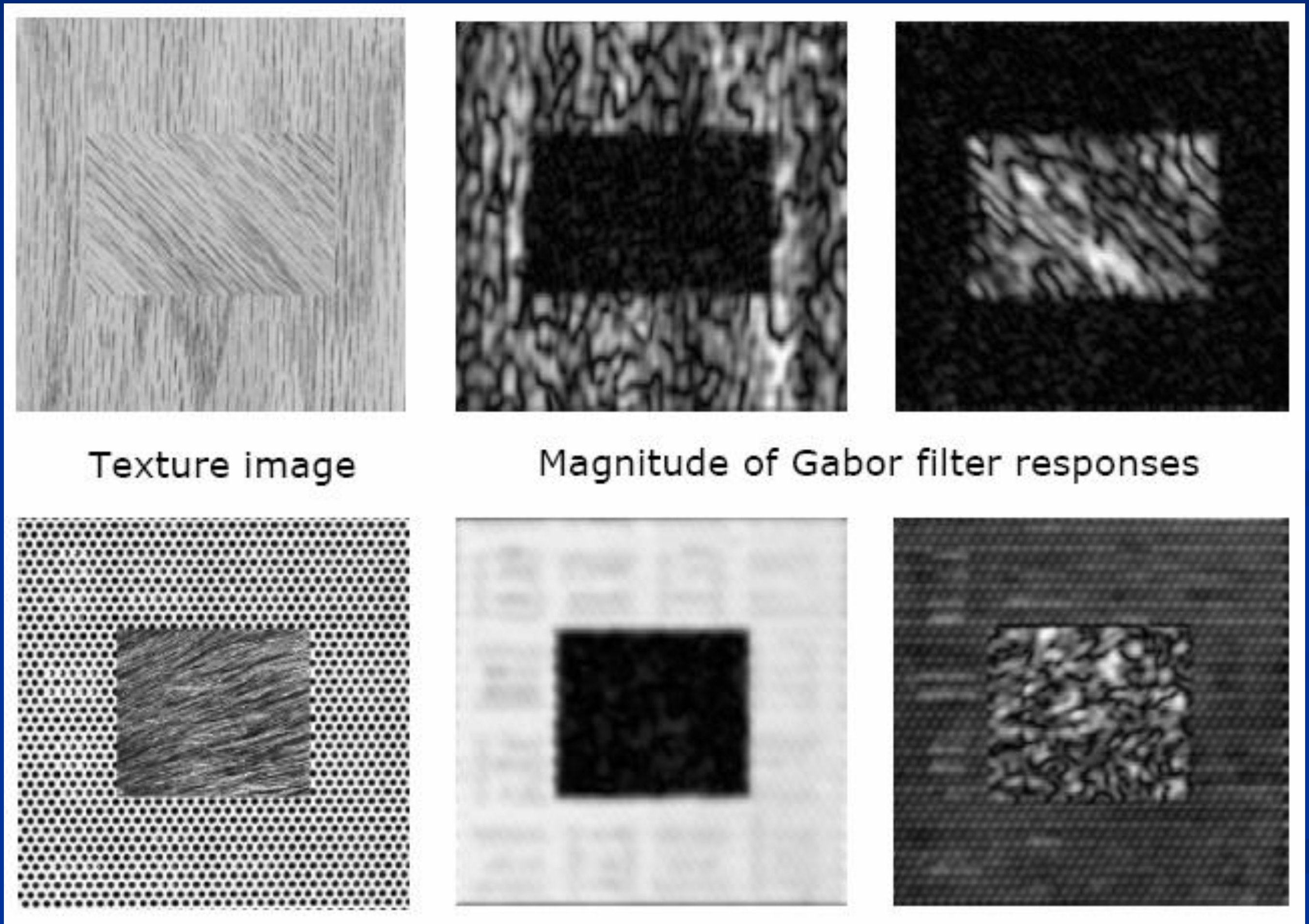
Банки фильтров



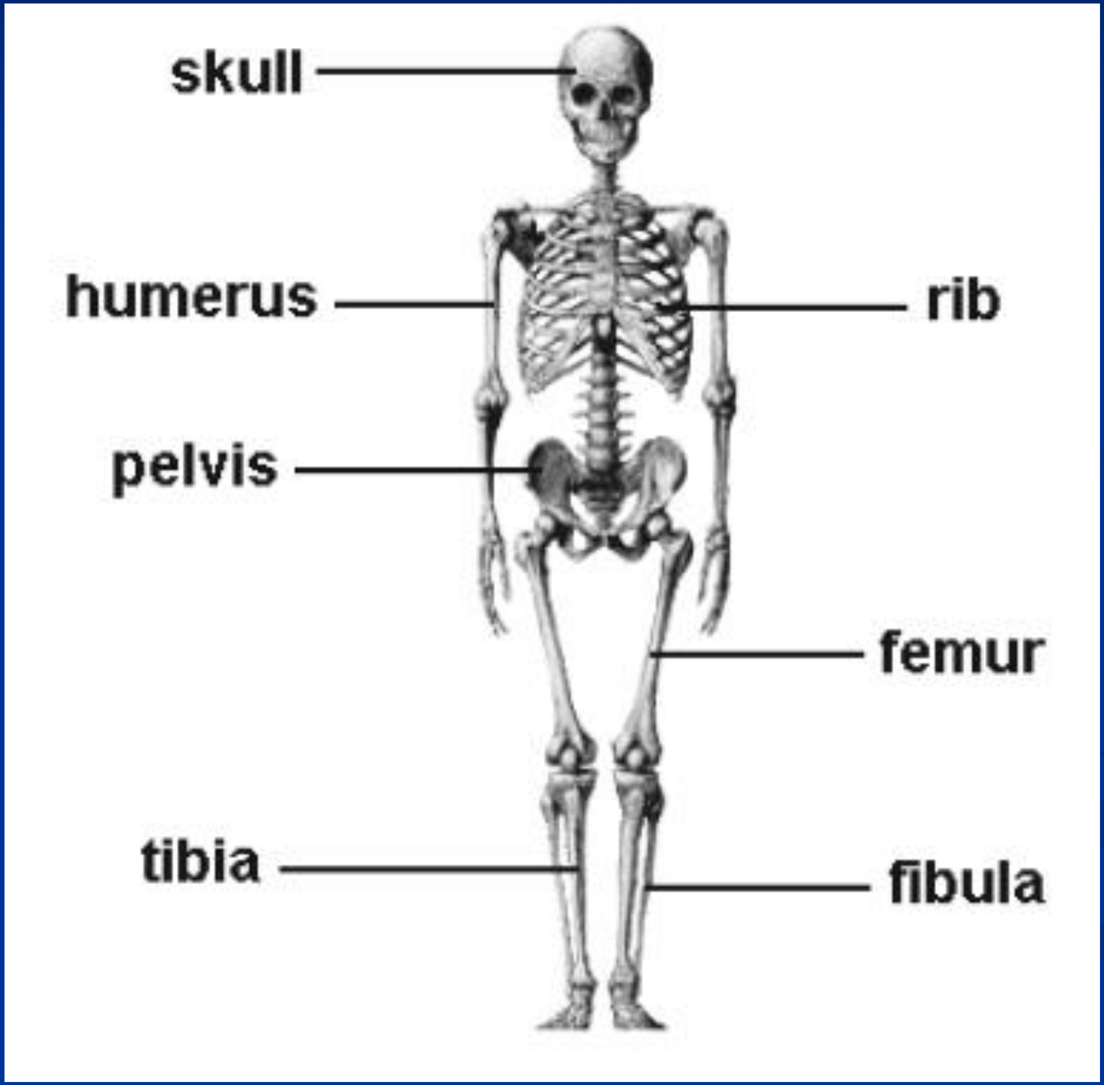
Банки фильтров



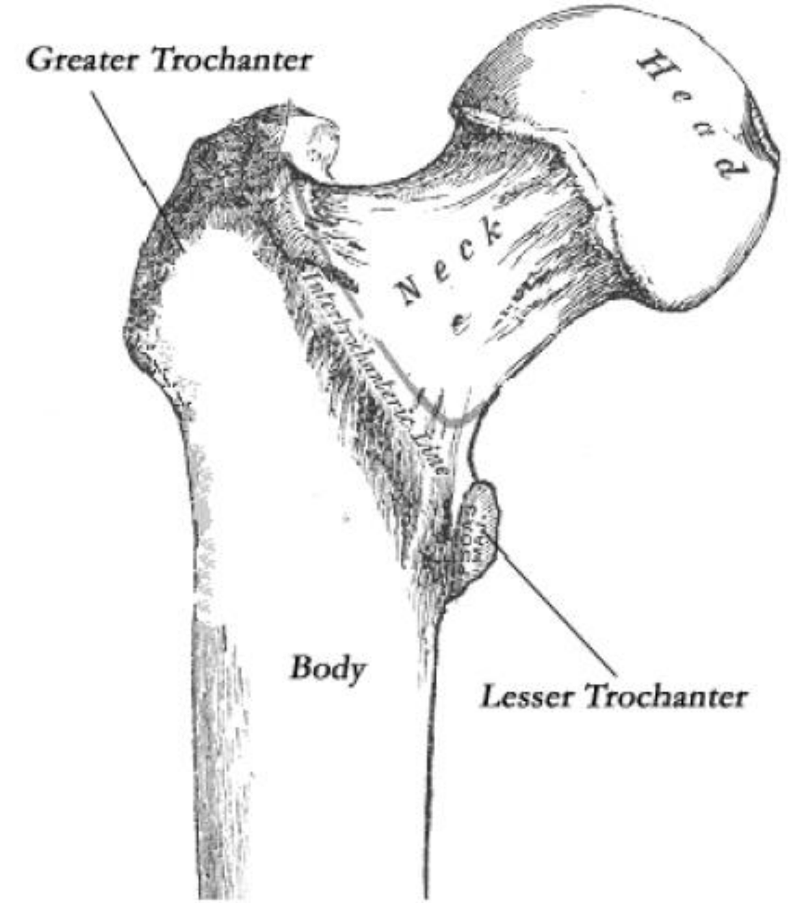
Фильтры Габора



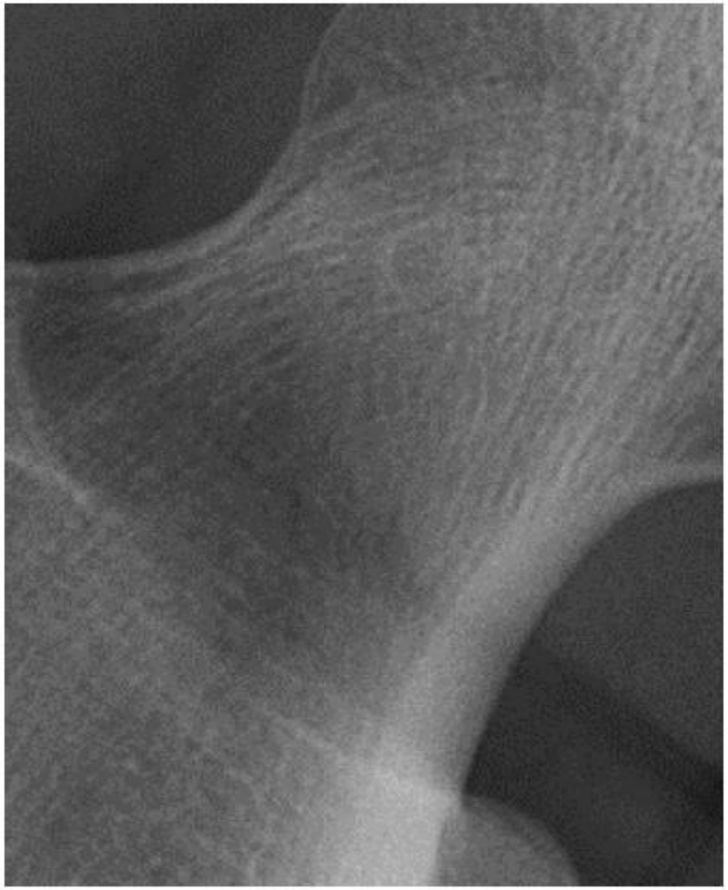
Анализ бедренной кости (femur)



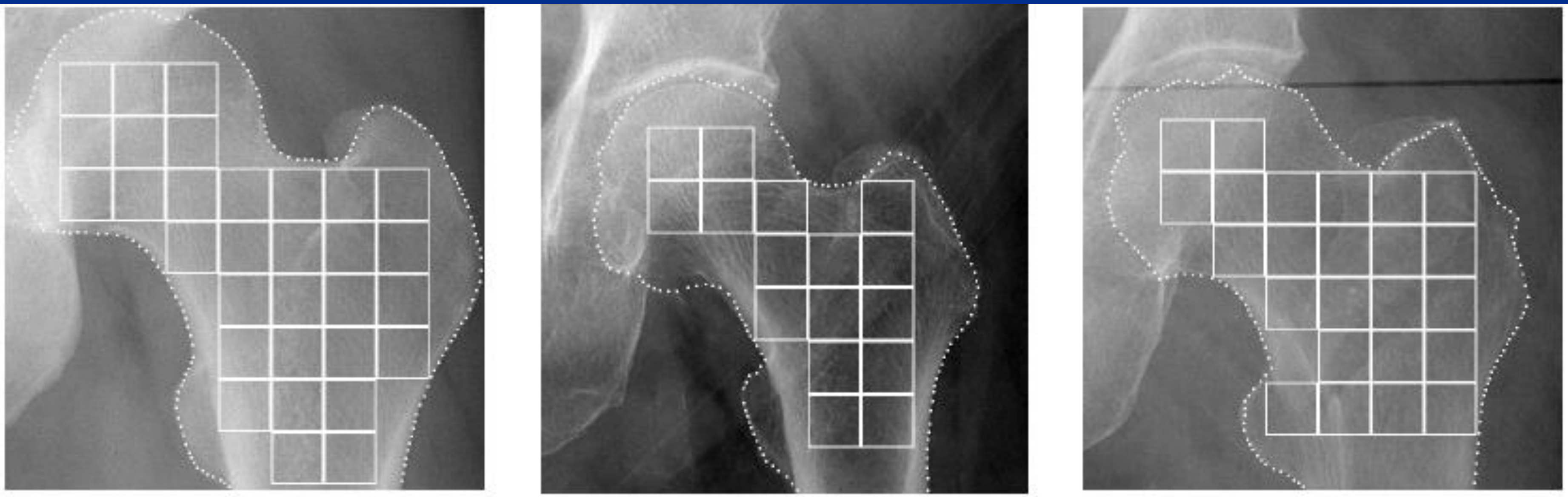
Шейка бедра



Greater Trochanter
Head
Neck
Intertrochanteric Line
Body
Lesser Trochanter



Шейка бедра



(a) (b) (c)

Figure 3.3. A femur is a natural structure that exhibits variations. (a) and (b) above are healthy, while (c) is fractured. Here, (a) is larger than (b). (c) has a relatively shorter neck, due to the fracture. Each grid square is a region sampled for texture orientation feature extraction

$$h(x,y) = g(x',y')\exp(2\pi jfx').$$

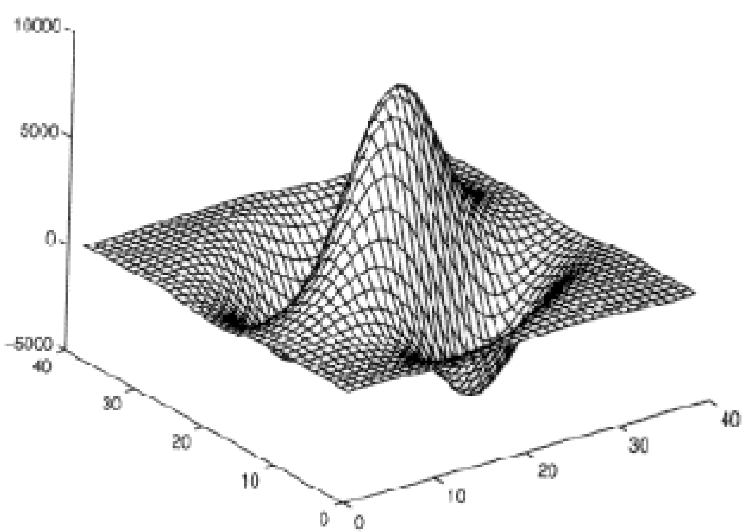
The oriented Gaussian function $g(x', y')$ is given by:

$$g(x',y') = \frac{1}{2\pi\lambda\sigma^2}\exp\left[-\frac{(x'/\lambda)^2 + y'^2}{2\sigma^2}\right],$$

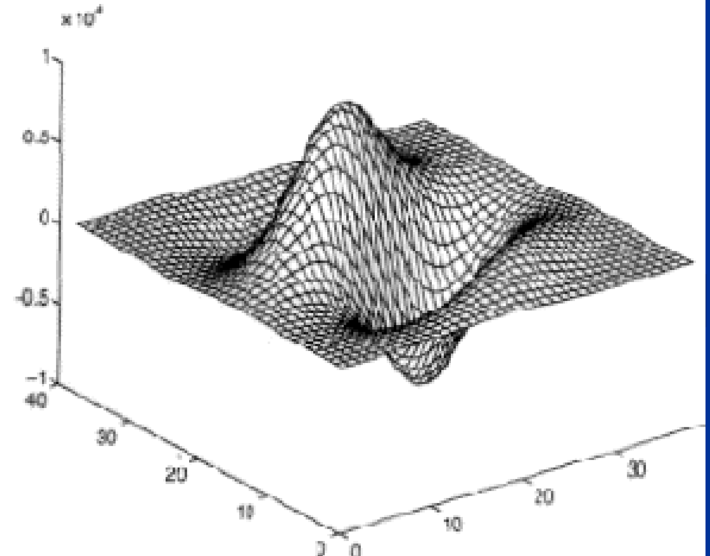
where $(x',y') = (x\cos\theta + y\sin\theta, -x\sin\theta + y\cos\theta)$ are rotated coordinates

$$h_{c,f\theta}(x,y) = g(x',y')\cos(2\pi fx'),$$

$$h_{s,f\theta}(x,y) = g(x',y')\sin(2\pi fx').$$

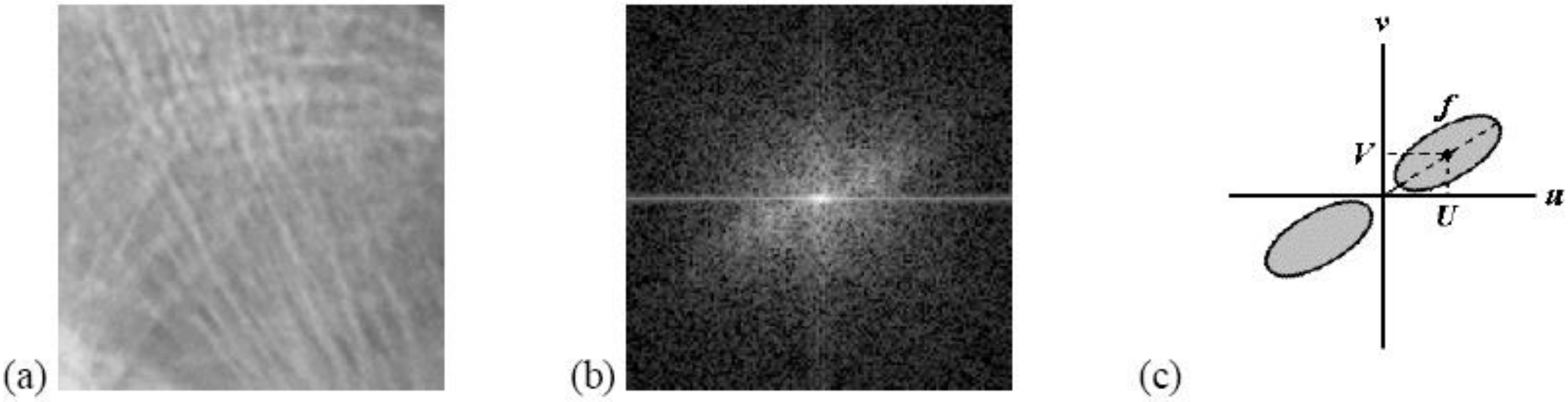


(a)



(b)

Фурье анализ текстуры кости



(a) (b) (c)

$$f_p = \sqrt{\overline{U}^2 + \overline{V}^2}$$

$f_p \cong$ 0.13 cycles per pixel

A Gabor filter bank of 1 frequency channel and 8 orientation channels is used to extract the orientation of the texture patterns in the femur. The centre frequency f of the Gabor filters is set to 0.13 cycles per pixel, and orientations range from 0° to 157.5°, incrementing in steps of 22.5°.

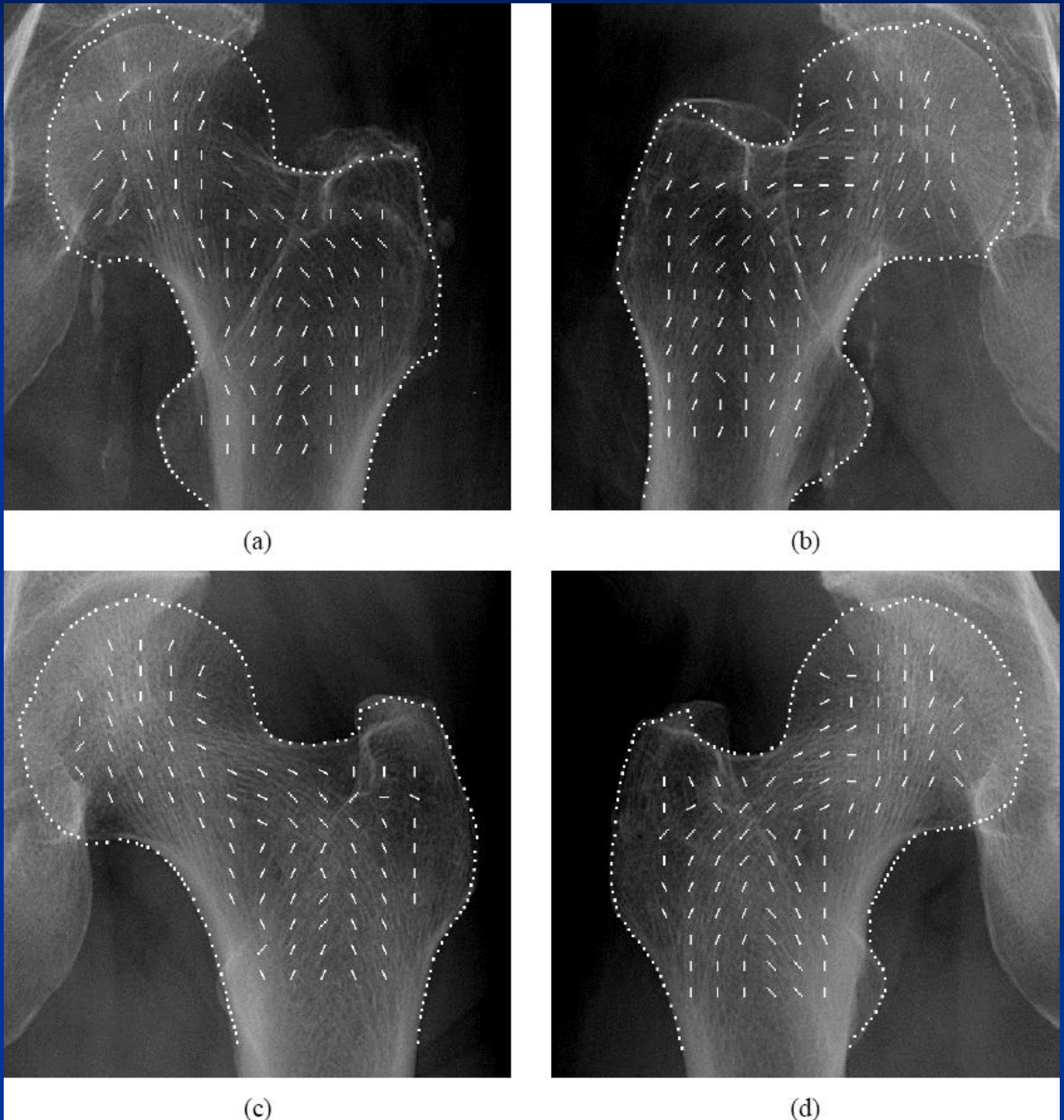
$$e_{c,f\theta}(x,y) = I(x,y) * h_{c,f\theta} ,$$

$$e_{s,f\theta}(x,y) = I(x,y) * h_{s,f\theta} .$$

$$E_{f\theta}(x,y) = e^2_{c,f\theta}(x,y) + e^2_{s,f\theta}(x,y)$$

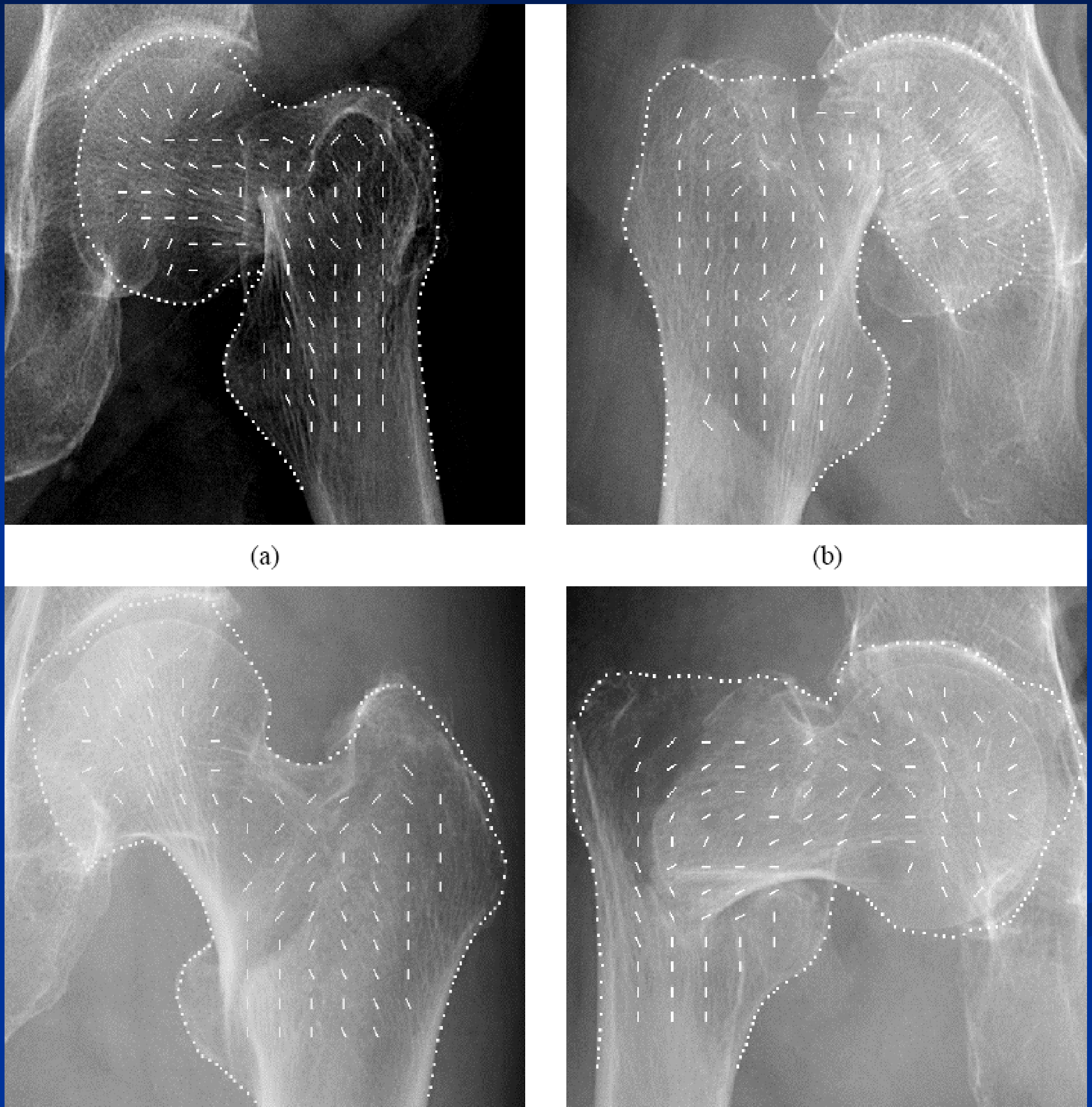
$$\overline{E}_{f\theta} = \frac{1}{S_x S_y} \sum_{x=1}^{S_x} \sum_{y=1}^{S_y} E_{f\theta}(x,y)$$

Целые кости

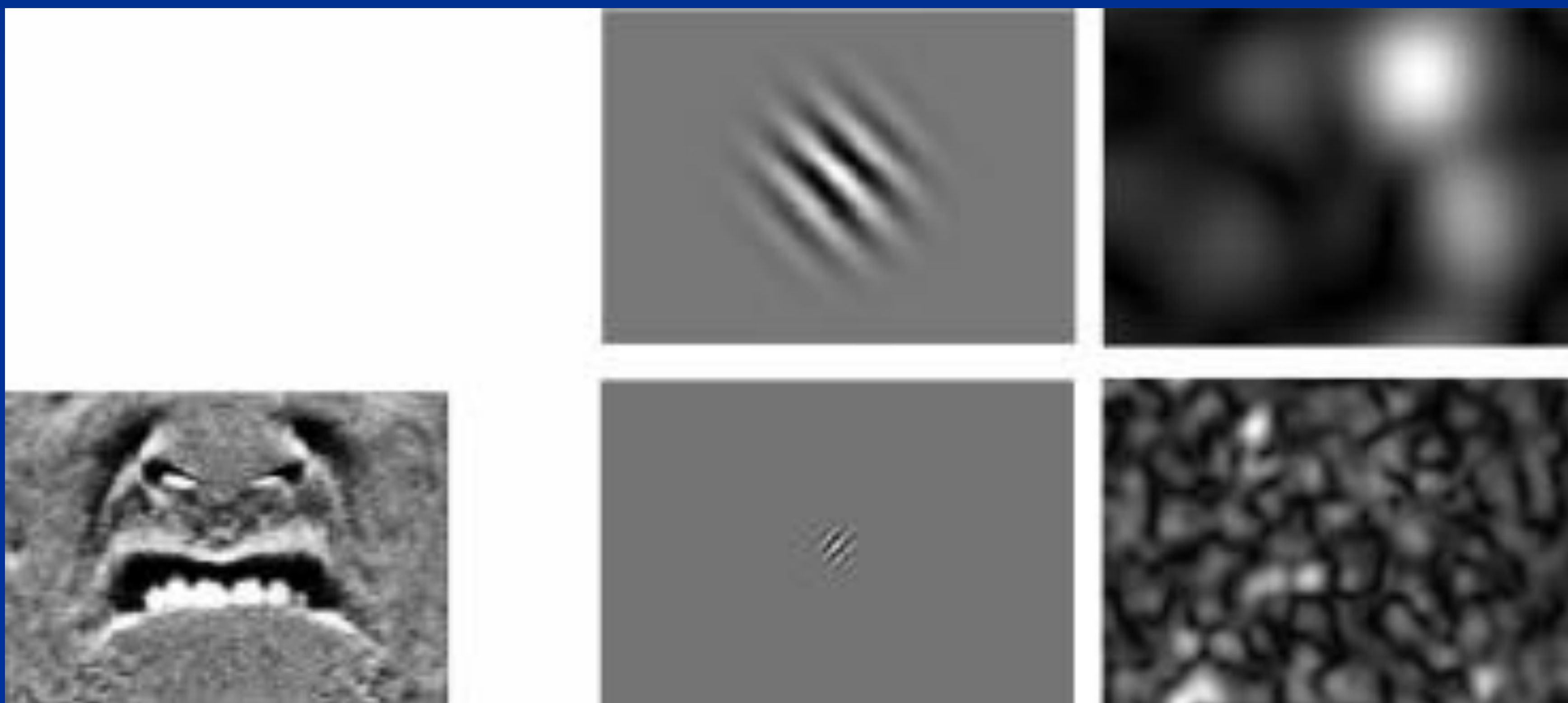


(a) (b) (c) (d)

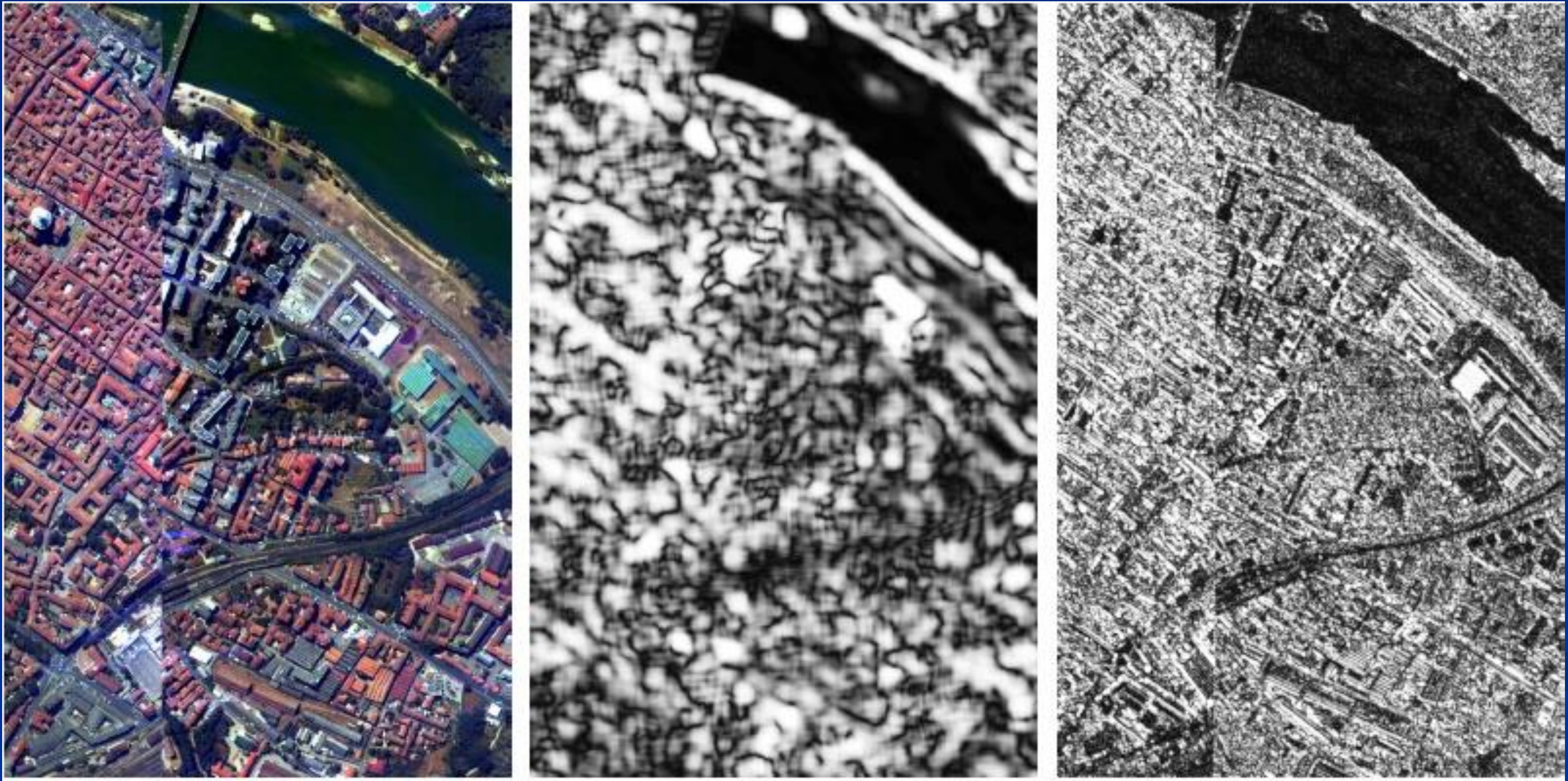
Кости с переломом



Фильтры Габора

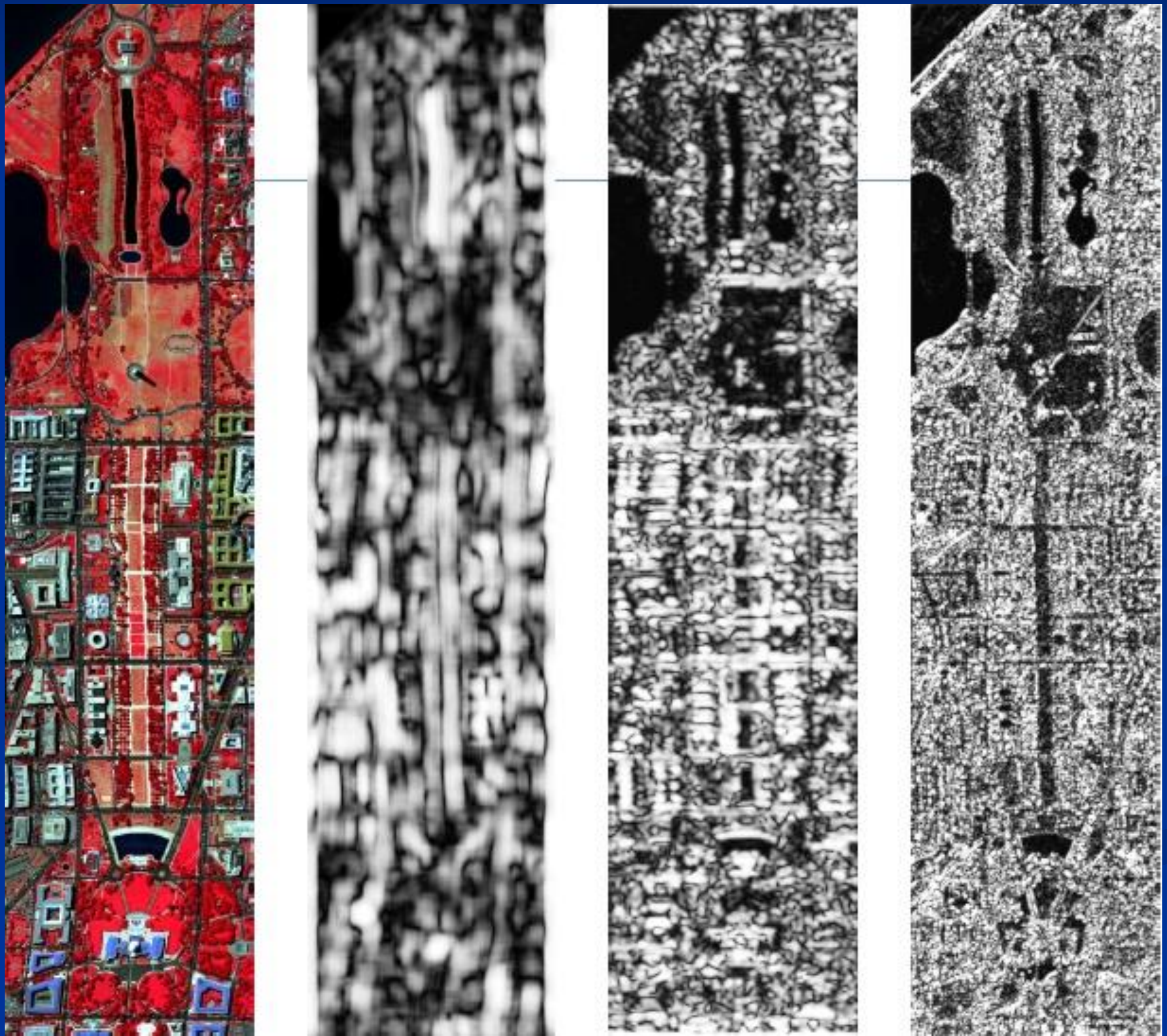


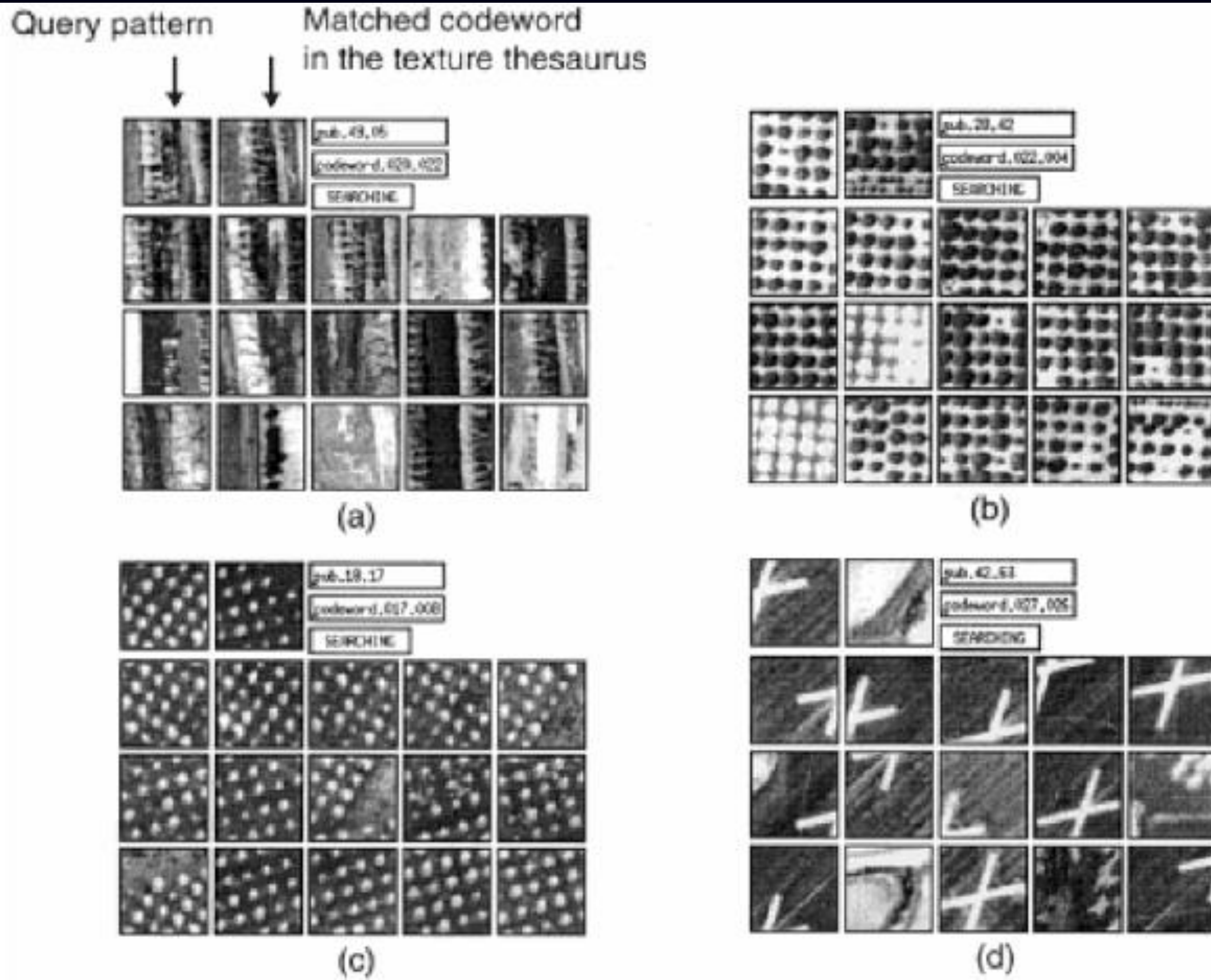
Фильтры Габора

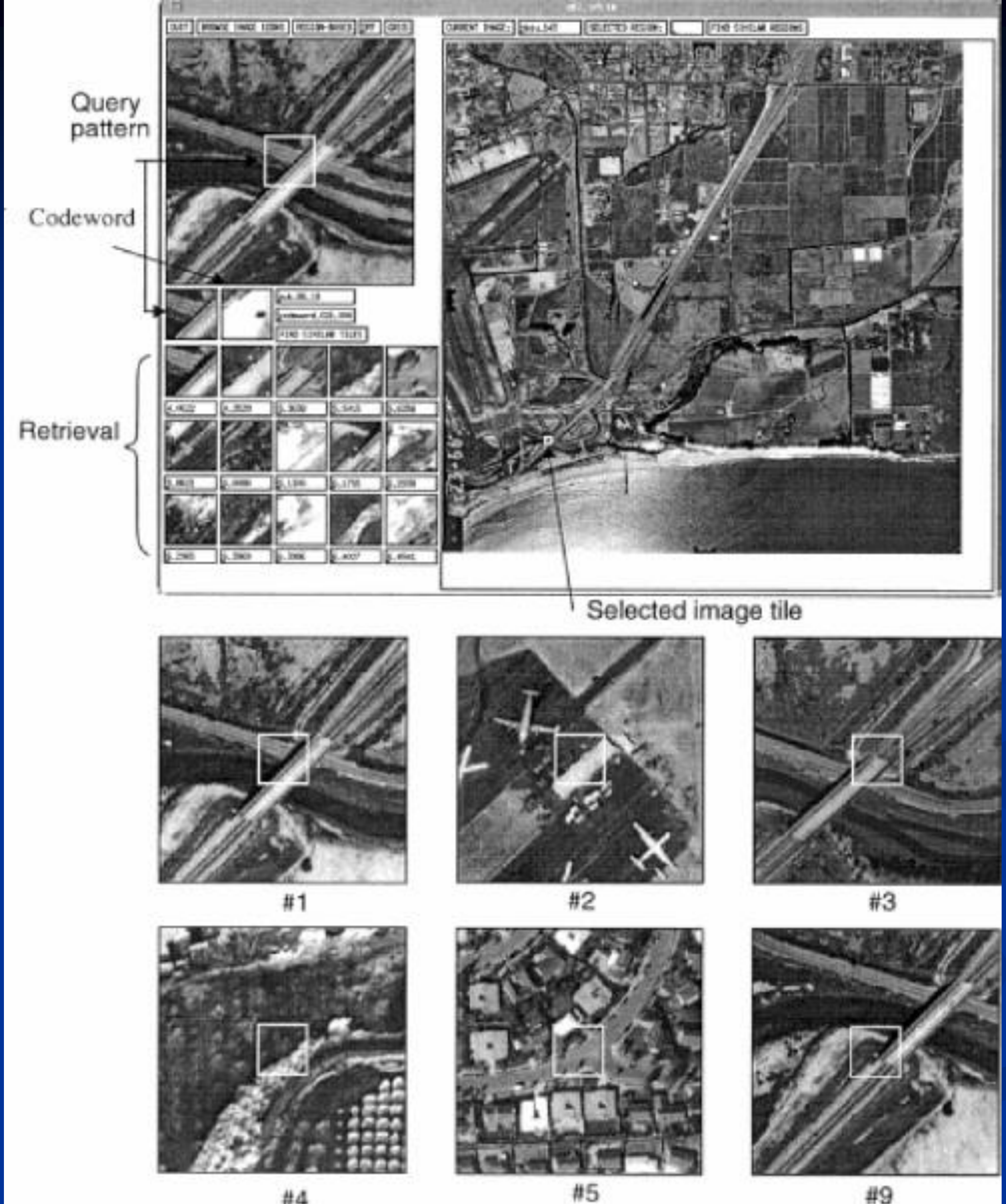


Gabor filter responses for a satellite image.

Фильтры Габора

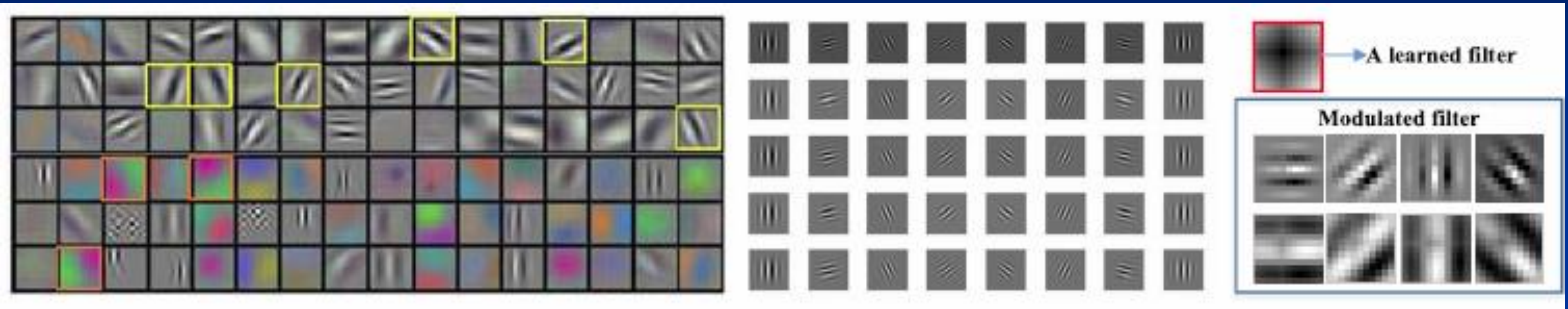



Query pattern
Matched codeword
in the texture thesaurus
(a)
(b)
(c)
(d)



Full resolution of some retrieved tiles

Gabor Convolutional Networks

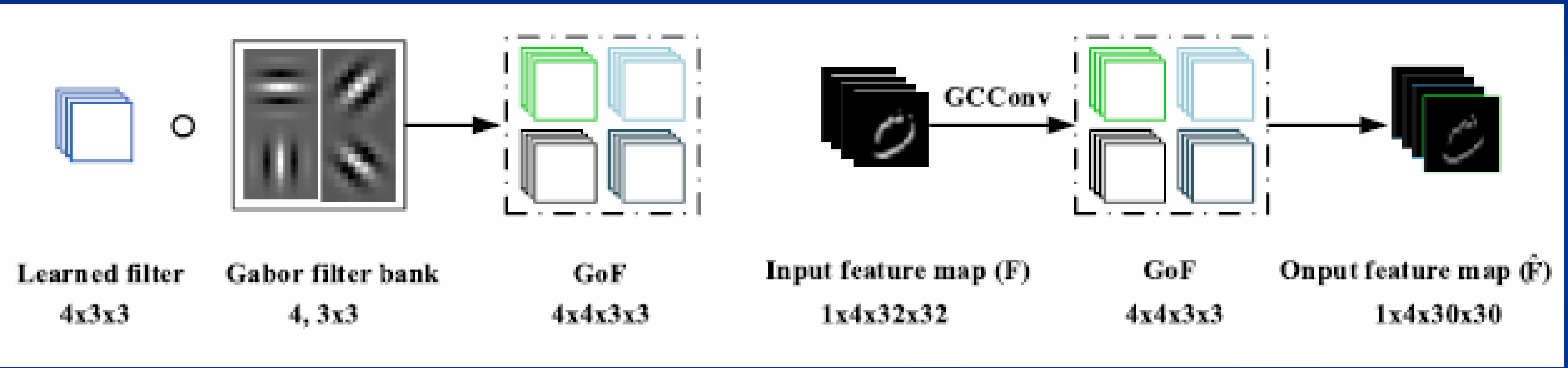


Left illustrates AlexNet filters. Middle shows Gabor filters. Right presents the convolution filters modulated by Gabor filters. Filters are often redundantly learned in CNN, and some of which are similar to Gabor filters (see the highlighted ones with yellow boxes).

Based on this observation, we are motivated to manipulate the learned convolution filters using Gabor filters, in order to achieve a compressed deep model with reduced number of filter parameters.

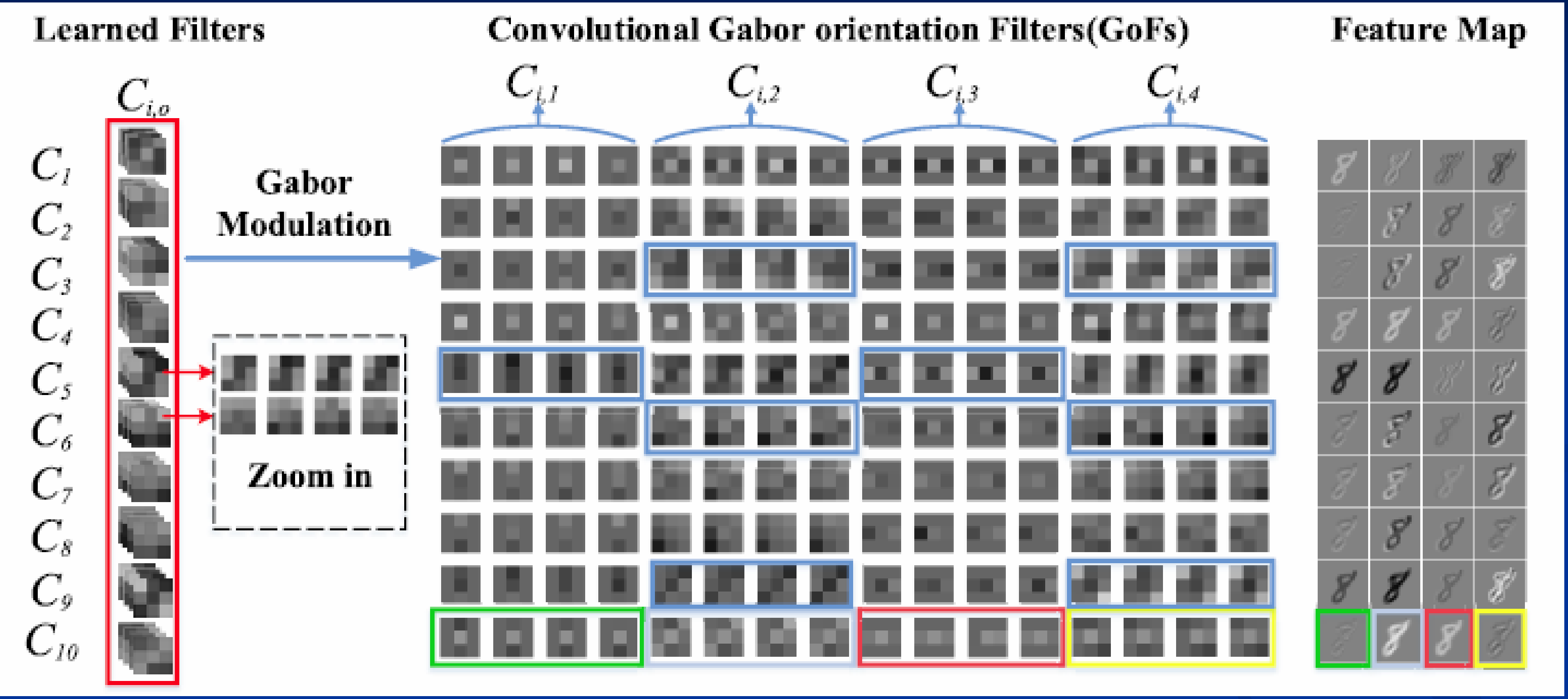
In the right column, a convolution filter is modulated by Gabor filters to enhance the orientation property.

Gabor Convolutional Networks



Left shows modulation process of GoFs.
Right illustrates an example of GCN convolution with 4 channels. In a GoF, the number of channels is set to be the number of Gabor orientations U for implementation convenience.

Gabor Convolutional Networks



Visualization of the first convolution layer of GCNs. Each row represents a group of GoFs and its corresponding feature map. i.e. the 10th GoFs (C10,1 . . .C10,4). Each 4-orientation channel GoF is labeled as different colors. The output feature map also has 4-orientation channel, which is labeled with the same color as its corresponding GoF.

The examples in the blue rectangle show that GOFs carry various orientation information.